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Modeling and Simulation of the Antikythera Mechanism

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Abstract

Technological advances have played a major role in the study of the Antikythera mechanism since the day of its discovery. From the radiographs performed by Dr. Karakalos, until the study of modern day Computer Aided Tomographies, this amazing device has been the application field of numerous new technologies. Computer modeling and simulation was one of the first techniques used for the analysis and study of the mechanism, and they were responsible for the sharp increase in scientific interest for its operation. This is attributed to the fact that simulations and computer models can be rapidly and easily distributed through the Internet. As a verification tool computer simulation can be used for the verification of a design be- fore the machine is actually built, or the analysis of an existing machine. In the case of the Antikythera Mechanism we are faced with a completely deterministic system whose operation is not known in its entirety, derived only by the parts of the mechanism that have been found and studied. Since it is considered by many to be the first portable computer ever built, it is very appropriate to use a modern date computer simulation for validating the functional description of the machine, as well as estimating the possible operation of missing parts. From identifying errors in Derek de Solla Price's drawings, to understanding the implementation of the Hipparchos lunar orbit observations, to calculating the exact tolerances required for the construction of a physical model, the simulation of the mechanism has been proven invaluable. Computer simulation has the additional advantage of being able to quickly verify a multitude of hypotheses as in the case of an as yet unidentified fragment that may have been part of a celestial observation display. Compared with a physical model that can only be used for demonstrating purposes, a simulation model can be easily calibrated to exact dates by computationally "turning" the gears of the mechanism to the required time. Then, it can be used for calculating astronomical phenomena in accordance to the mechanism's original design, without the need for cumbersome adjustments.

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1. Introduction

The first models of the Antikythera Mechanism ever built were physical models and they were built based on the findings of Derek de Solla Price. Derek de Solla Price studied the Antikythera mechanism for 25 years during which he published his papers of 1959 [4] and 1974 [5]. Although quite a few researchers before Price presented various hypotheses about the mechanism, it was Price who was able to meticulously count the teeth of each gear based on the radiographs of Dr. Karakalos, and give the first detailed description of the mechanism's operation. Today, based on the newer finding of the Antikythera Research Project [2], we know that some of his conjecturers, like the differential turntable, are incorrect. Nevertheless, Price built the solid foundation on which any further research on the mechanism is based.

Price is shown in Figure 1 on the left with the physical model he built. Neither his model, nor most of the physical models that were created subsequently operate flawlessly. This is mainly attributed to the fact that most of the gears' bearings cannot be clearly distinguished in the radiographs or computer aided tomographies and, to be able to clearly display the inner gears, most reconstructions make the mounting plates less robust than they should be [1]. The only, until now, physical reconstruction that operates flawlessly is the one made by Michael Wright [11] but it slightly deviates from the original design by adding some mounting plates. Wright's model is shown in Figure 1 on the right. Other more recent operational physical models demonstrate only the functions of the mechanism, without being accurate reconstructions.

Although physical models were built early in the study of the mechanism, simulation models had to wait for the advancement in both computer hardware and software tools before they started to emerge. The first computer simulation was developed by Robert Morris [3]. He used a block diagram compiler to create an animated version of the mechanism's gears for a vector graphics system. This was an experimental model and not a real simulation of the mechanism, since at the time of its development (around 1980), graphics terminals were not readily available.

Figure 1: Physical Models of the mechanism, Price's (left) and Wright's (right).

This paper examines accurate computer modeling, simulations and animations of the Antikythera mechanism. The study takes into account purpose, scope, availability, and accessibility of the models, to demonstrate that simulations provide an invaluable tool for understanding the mechanism's operation and for the dissemination of the research's results.

1.1.Modeling and Simulation

Modeling refers to the creation of a representation of a system or phenomenon. It may even refer to the complete amount of information we have collected for the study of a system. Models are either physical or mathematical and can be classified according to the taxonomy shown in Figure 2 [7].

Both physical and mathematical models are subdivided into static and dynamic depending on whether they change in time or not. Then, mathematical models are further subdivided into analytical and arithmetic models depending on whether there exist analytical equations to describe them or not. As shown in this figure, simulation is mainly used for arithmetic, dynamic, mathematical models.

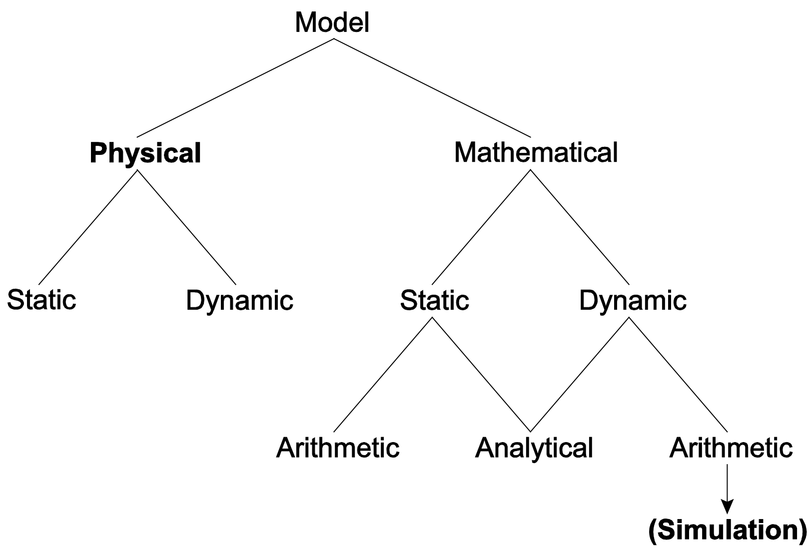


Figure 2: Classification of models.

Based on the models' taxonomy, it would seem more appropriate to build a physical model for a deterministic mechanical system like the Antikythera Mechanism, rather than a computer simulation. However, the classification refers only to system analysis or synthesis and not to other uses of models, like education, where they are used to demonstrate the complex operation of a system through direct visualization. This is the reason we frequently see animation and simulation models of engines, machines, and devices.

In addition, as will be shown in the following paragraphs, apart from its obvious educational value, computer simulation of the Antikythera Mechanism has contributed quite a lot, mainly to the validation of the mathematical models and hypotheses, the identification of errors in the physical designs, and the verification of operational descriptions.

2. Verification of Price's conjectures

The functional description of the Antikythera mechanism as derived from the study by de Solla Price included detailed drawings of the mechanism. To verify these drawings and visualize the operation of the mechanism we tried to create the first completely operational computer model of the mechanism. The transformation of the drawings into computer aided design revealed a error in the drawings as demonstrated in Figure 3.

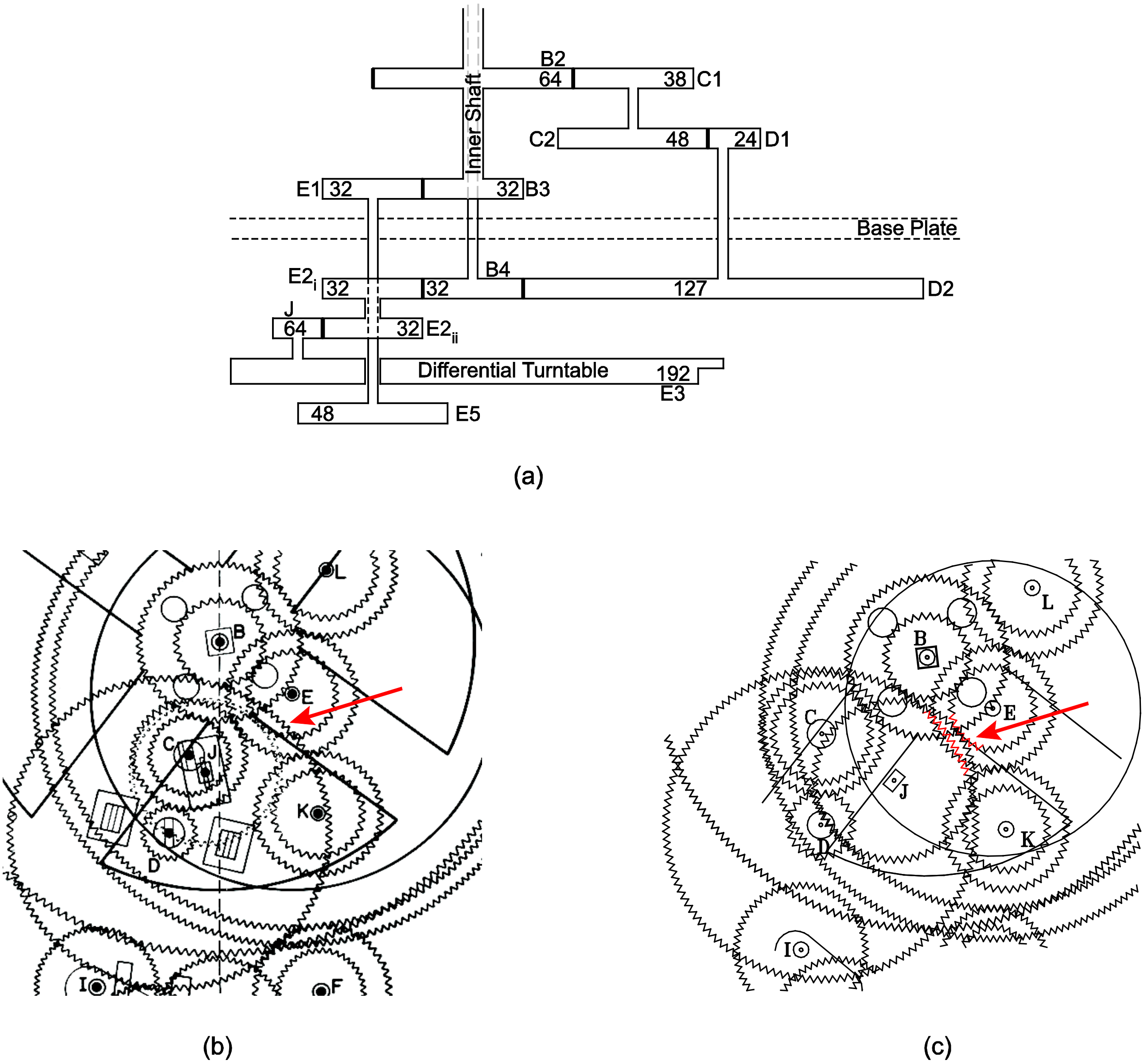


Figure 3: Recalculation of Price's gear positions.
(a) Part of the sectional diagram.
(b) Part of Price's planar view,
(c) Repositioning of the gears.

The drawing on the top (Figure 3(a)) is part of the sectional diagram as given by Morris in [3]. As can be seen, gear E2i engages gear B4 which engages gear D2. It follows that gears E2i. B4, and D2 are on the same plane. However, in the planar diagram shown in Figure 3(b), gears E2i and D2 overlap at the position pointed by the red arrow. As this cannot possibly be the case, the design was corrected for the computer simulation model, by slightly modifying the position of the gears' axes. In the final model, the gears have quite some clearance as shown in Figure 3(c) by the red arrow and the red gear teeth.

Based on the modified drawings, the first computer generated animations of the mecha- nism was created in 1998 [9] and published in a multimedia CD [8]. These animations used pseudocolors for the gears, to make the operation of the mechanism more comprehensible. Two frames of these animations are shown in Figure 4.

In addition to static representation of the model's dimensions, computer simulation models of the Antikythera Mechanism require the accurate specification of either rotation equations for the various parts (mainly gears), or the generation of accurate positional data for each subpart. As these specifications are based on the physical measurements of the mechanism, they can be used to verify the obtained data. As an example, consider the simulation modeling of the Hipparchos gear train [2]. The rotation of gear k1 (the one with the pin) can be modeled simply be the relative rotation of the gears driving k1. However, the rotation of gear k2 is rather complicated. The best way to model it for the simulation, is to calculate the rotational angle of gear k2 relative to the rotational angle of gear k1. Figure 5 shows the sectional diagram of gears k1 and k2 (Figure 5(a)) and the calculation of angle θ' as a function of angle θ.

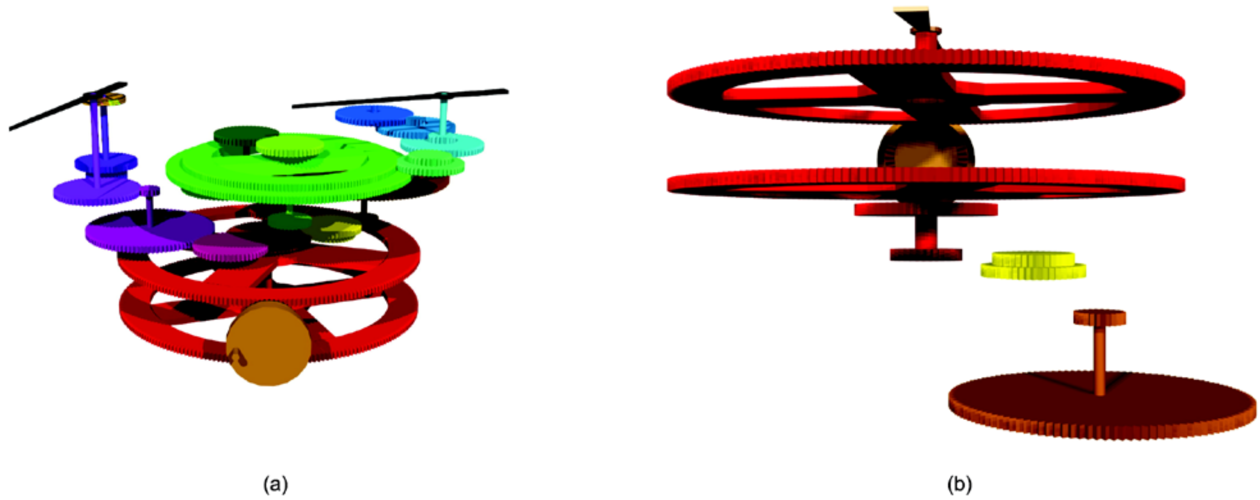


Figure 4: Two frames from the original animations. (a) A frame showing the differential gear at the top. (b) A frame showing the assembly of the mechanism by the gears falling into place.

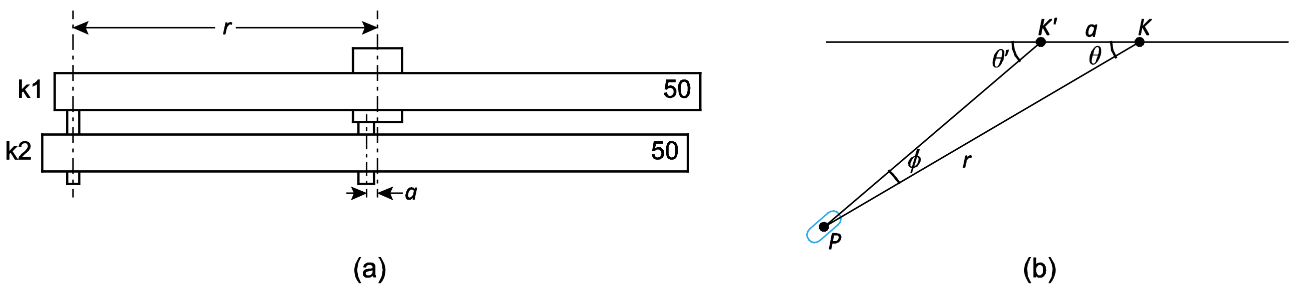


Figure 5: Calculation of the relative angle of the slotted Hipparchos gear.

For the triangle shown in Figure 5(b), K is the center of gear k1, K′ is the center of gear k2, and P is the pin attached to gear k1. Thus, sides KK′ and KP of the triangle are of constant length (a and r respectively), while side K′P is variable, depending on the position of the pin inside the slot. The sine law states that:

$$\frac{r}{\sin(180-\theta')}=\frac{\alpha}{sin\phi}\Longrightarrow sin\phi=\frac{\alpha}{r}\sin(180-\theta')=\frac{\alpha}{r}\sin(180-\theta-\phi),$$

Using the sum of angles formula

$$sin\phi\left(1+\frac{\alpha}{r}\cos(180-\theta)\right)=\frac{a}{r}\sin(180-\theta)\,cos\phi$$

and setting

$$x=1+\frac{\alpha}{r}\cos(180-\theta),\text{και } y=\frac{\alpha}{r}\sin(180-\theta)$$

we get

$$sin^2\phi x^2=y^2(1-sin^2\phi),$$

from which we finally get

$$sin\phi=\frac{y}{\sqrt{x^2+y^2}},\text{ή } \phi=\arcsin\left(\frac{y}{\sqrt{x^2+y^2}}\right),$$

Since $\theta'=\theta+\phi$ the relation of θ' and θ is

$$\theta'=\theta+\arcsin\left(\frac{y}{\sqrt{x^2+y^2}}\right).$$

Thus, in simulating the rotation of the various gears, the position of gear k2, as defined by angle θ' can be computed by the position of gear k1 as defined by angle θ.

Another use of simulation modeling for verification of the mechanism's operation is the computation of the gear parameters. Both the old radiographs and the modern tomographies showed that the teeth of the Antikythera mechanism's gears were triangular. This teeth geometry is the easiest to manufacture, especially with the tools available at the time, but the least appropriate for smooth interlocking. The reason is clearly shown in Figure 6. If the distance of the gear centers is such that the tip of a tooth falls very close to the land between the teeth, then the gears cannot rotate because the tip hits the side of the opposite tooth.

Depending on the actual inner (at the lands) and outer (at the teeth) diameters of the gears, what needs to be computed is the pitch radius of each gear. The pitch radius is defined as the radius at which two cylinders would roll on each other, assuming that the gears are replaced by touching cylinders. Taking into account that the cylinders cannot slide at all, the pitch ratio is exactly equal to the gear ratio. Thus, by specifying one of the pitch radii, the other one can be directly derived. If the clearance, defined as the distance between one tooth's tip from the opposite land, is too small, rotation is not possible as explained before. If the clearance is too large, then there is inadequate transfer of torque between the gears, resulting in a difficult or impossible operation of a long gear train. This is probably the reason that most of the physical models that are accurate reconstructions of the Antikythera Mechanism, do not operate smoothly if at all. On the other hand, computer simulation models can operate smoothly due to the precise calculation of the clearance between each pair of interlocking gears.

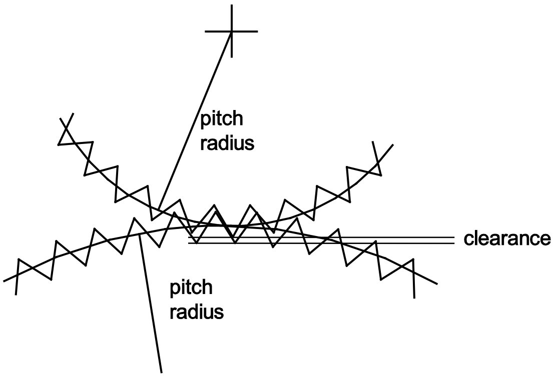
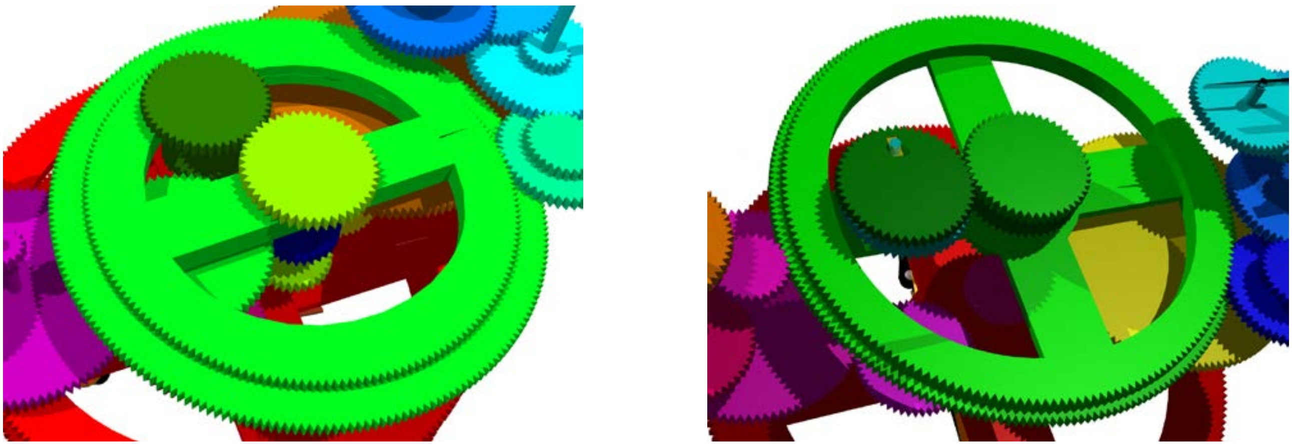


Figure 6: Gear parameters.

The computer aided tomographies performed by the Antikythera research project contradicted Price's placement of a differential gear. Apart from being a very complicated gear-train for a relatively simple derivation it did not account for the slotted K2 gear and fact that such a large gear like E3 would have to turn at a very fast speed, requiring excessive force on the driving crown. The CAT scans revealed that the complicated gear-train which was assumed to drive the differential gear, is in fact the modeling if Hipparchos' lunar anomalies computation. Based on those facts, the computer modeling was very easily adjusted as shown in Figure 7.



(a) (b)

Figure 7: Replacement of the differential gear. (a) Price's differential gear (b) Hipparchos' gears in the new model.

3. Modeling for Hypothesis Testing

Since the main purpose of developing the simulations and animations of the Antikythera Mechanism was the validation and verification of the measurements obtained by either de Solla Price or the newer research, we tried to avoid the inclusion of any missing parts of the mechanism. This was not always possible, because the operation of some parts cannot be demonstrated without showing their interconnection to other parts. However, the insertion of missing parts for the interconnection was kept to the simplest form possible for two reasons: a) the mechanism is very elaborate and elegant, which implies that its craftsman would have gone to the simplest solution possible for any design problem he faced, b) a complicated solution may inadvertently include some engineering knowledge that was not available at the time of the mechanism's development.

This insistence for simplification of the missing parts forced us to include inaccuracies to the simulation rendering, instead of including complicated gear trains. This is illustrated by the rendering of the moon pointer's mounting on its axis, as shown in Figure 8. Only the crown shown in the figure was identified in the existing fragments of the mechanism. The small gear on the sun pointer's axis was not found. The problem is that the crown is, in reality, facing the other way around. This would require the inclusion of at least 3 more gears and a subframe to support them, in order to convert the horizontal rotation of the two pointers into the relative vertical rotation of the moon's sphere that depicts the moon's phases.

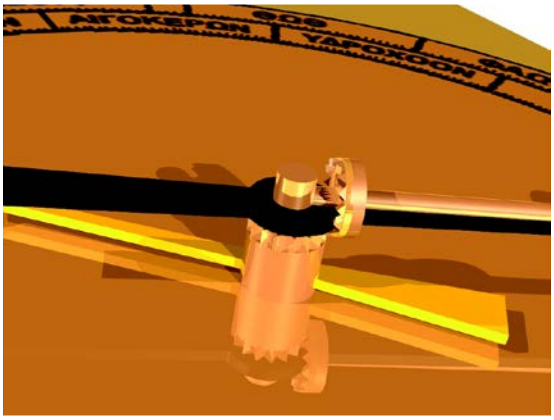
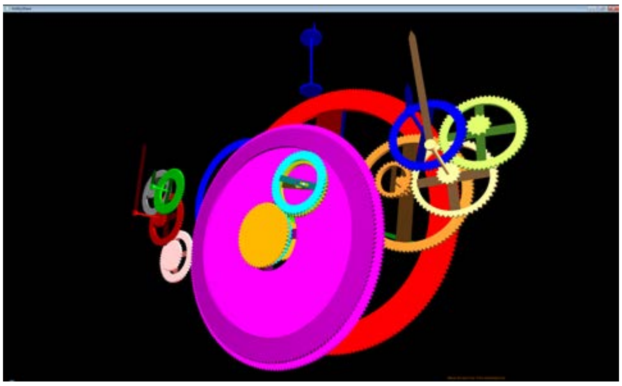


Figure 8: Rendering of the moon pointer mounting.

The solution included in the simulation models is extremely simple and elegant, fol- lowing the philosophy of the mechanism, even though is does not exactly correspond to the available fragments of the mechanism.

4. Simulations for detailed study and operation verification



(α)

(β)

Figure 9: Simulation of the mechanism. (a) View from the back, showing the Hipparchos gears. (b) ”Zooming-in” to check clearance of the gears.

The modeling of the Antikythera Mechanism presented so far produced a series of animations, which demonstrated the operation of the mechanism in various ways. However, to more precisely study the operation, a number of actual simulations was developed. The first set of simulations give the user complete control of the model, so that the user can zoom in, rotate and move the model, in order to examine it from every possible angle. Since the model is described by vectors, the detail is present regardless of the amount of zoom applied. Therefore, minute details of the gearing can me examined to determine the exact meshing of the gears, any discrepancies in their movement, or any problems with the rendering. At any given position, the user can simulate the turning of the crown, thus putting the mechanism in motion and observing its operation. The simulations of the Antikythera Mechanism are posted on the same site [9], again being freely downloadable. Two frames of the simulation are shown in Figure 10.

In addition to their obvious value for scientific research, the simulations are also an educational tool to demonstrate the mechanism’s operation in museums or publications. For example, the simulations have been used at an exhibition entitled ”Gods, Myths, and Mortals: Discover Ancient Greece” hosted by the The Children’s Museum of Manhattan in 2007-2008. Also, a composite picture of the animation appeared on the cover of engine [10] a magazine for teaching English to German engineers!

These simulations cannot be used to check the accuracy of the mechanism’s calculations over o long period of time. For this reason, a more abstract simulation model of the mechanism was constructed, that can be used to rapidly ”operate” the mechanism until it reaches a certain date. The user enters the date and the simulator computes the position of the four cycles calculated by the Antikythera mechanism, namely the Metonic, Callipic, Saros, and Exeligmos cycles. The computation is not performed by solving equations, but by ”turning” the gears of the mechanism as many times as required to reach the date set by the user, and then ”reading” the back dials of the mechanism. Such a simulated computation is shown in Figure 10.

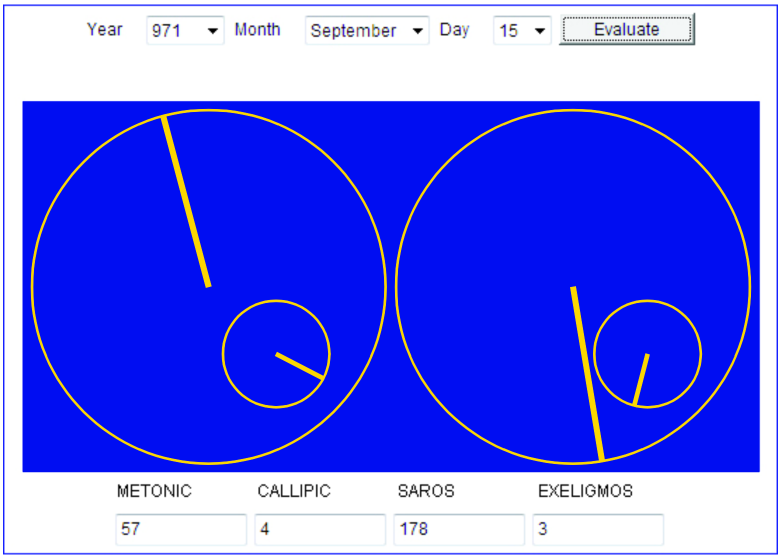


Figure 10: Simulation to compute the various astronomical cycles.

5. Conclusions

This paper demonstrated how computer modeling and simulation can be used to analyze and verify ancient artifacts, and in particular, the Antikythera Mechanism. Simulation can be used to verify the estimated parameters of the mechanism and validate various hypotheses about the mechanism's design. Compared to the creation of physical models, computer modeling has the distinctive advantage of accurate and rapid evaluation of functional descriptions. Also, computer simulation models, being easily distributed, provide widespread awareness, greatly enhancing the appreciation of Ancient Greek technology. Finally, simulation can be used to emulate certain aspects of the mechanism's operation, giving us the opportunity to ”use” it even today.

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