Kepler meets Alexander the Great: the Hellenistic legacy in Astronomy

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Johannes Kepler meets Alexander the Great: the Hellenistic legacy in Astronomy and its history through the ages and through the Civilizations

The time interval between the era of Johannes Kepler and the Alexandrian Science in general, and the Alexandrian Astronomy in particular, spans for many centuries, and it might seem that these two great epochs of vivid intellectual activity have nothing in common. After a careful inspection, on the other hand, we can conclude that these two "noospheric toposes" are closely interrelated and interconnected [1], whereas the Alexandrian epoch can be regarded as the fountain that gives birth to the era of the famous astronomer, and shapes considerably his thought and his work [2].

The scientific work of Johannes Kepler

Johannes Kepler, as a personality, is a multi – dimensional intellectual existence, where opposites meet and complement each other. He is a deeply religious person, and his refugee to the philosophical speculations accompanies him for the whole of his life. At the same time he exhibits characteristics that place him among the Pantheon of great scientists, among Archimedes, Hipparchus, Claudius Ptolemy, as well as Galileo Galilei, Newton and Leibniz. He is governed by a strict epistemological empiricism, and believes in one, unique and real existence of Cosmos, which exists independently of the Mind of Man. He also believes strongly that Man, by his scientific inquiry, can penetrate into to the Mysteries of the Universe, and state them in the form of Physical Laws.

Johannes Kepler is mostly known by his three eponymous Laws of motion of the planets on their trajectories around the Sun. The first law determines the shape of the orbits of the planets, which is an ellipse, with the Sun situated at the one focus of the ellipse. The second law is of dynamical nature, and states that the ray starting from the point of location of the Sun and ending at the point of position of the planet traces equal areas in equal times. The third law is also of dynamical nature, and correlates the magnitude of the semimajor axes of the ellipses of the planets with their period of revolution in such a manner, so that the resulting ratio is equal for all the planets [3].

Kepler also contributes greatly to abstract Mathematics as well. He is one of the first to use extensively the calculus of the logarithms, as introduced by the work of John Napier [4] in his calculations for the Rudolphine Tables [5], he studies the properties of some forms of the so -called Kepler - Poinsot polyhedra [6], introduces a novel method, inspired by Archimedes, for the calculation of plane areas, and of surfaces of revolution [7], and invents a new periodic tiling of the plane through his “Monster” pattern [8], while he also states his Conjecture about the most dense packing of equal spheres in space, a conjecture proved only at the end of the 20th century [9].

Kepler becomes also famous as the first scientist to introduce for the first time the modern formulation of the science of Optics [5], and invents a more appropriate and useful form of the Galilean telescope, which is also a refracting telescope, but makes use of a different kind of lens, known as the Keplerian telescope [10].

All these great achievements of this marvelous genius are enough to place him in the Pantheon of the great Minds of civilization, among Archimedes, Hipparchus, Euclid, Ptolemy, Galileo, Poincare and Einstein.

The forerunners of Johannes Kepler

Both of these aspects can be traced within the Hellenic Alexandrian tradition of Science and Philosophy. The first refers to the development of the Neoplatonic and Neopythagorean tradition, and the second refers to the scientific spirit that prevails and expands in this great period of the human Civilization. These two aspects are conveyed by the life and work of many outstanding personalities, and we can mention briefly some of them, in a list that contains the names of Heron of Alexandria, Ctesibius, Eratosthenes, Aristarchus of Samos, Archimedes, Apollonius of Perga, Claudius Ptolemy, Plotinus, Proclus, etc. [11]

The Neoplatonic influence on Kepler’s thought

The neoplatonic influence on Kepler’s thought is more than evident. He always mentions in his work the great Nicholas of Cusa, whose thought and metaphysical, as well as ontological and epistemological views play the role of pillars in the great Astronomer’s attitude towards the Universe [12]. The work of Nicholas of Cusa, on the other hand, is based on the longstanding tradition of Neoplatonism [13], and he uses this tradition in his own idiosyncratic way [14].

According to the doctrine of the great Cardinal, Cosmos is an organic whole, a multilayered entity, with all its parts interacting with each other. Cosmos is the Image of the Nous of the Creator of the Universe, and inherits in the Form of Platonic ideas all its characteristics.

These characteristics appear in the form of Harmonies and Symmetries, both of Geometrical and Number Theoretic (Arithmetical) nature, and can be conceived by the human spirit after close inspection and careful measurement. Although the human mind cannot entirely, and will not, understand Nature and the Mystery of Creation in its totality, it can unravel the secrets of Cosmos by discovering the hidden relations between the parts that constitute its existence [15, 16, 17].

Nicholas of Cusa rejects the notion of perfection within the material Universe, hence the existence of perfect cyclic motions around the Sun, and the perfect crystalline celestial spheres that carry the planets on their motion around the Sun. The Earth cannot be the center of the Universe, since within the Universe a certain point that serves as its center cannot be defined. The Universe is the Image of the Nous of God, its Creator, and God can be defined as a circle whose center is everywhere, and whose perimeter is infinite [14].

Here, it becomes more than evident that these bold speculations served as the seeds for Johannes's Kepler thought, and combined with the heliocentric cosmological Pythagorean model of Cosmos, served as a profound basis for his long – life work [18].

The heritage of the Scientific Method and Kepler's achievements

Also, there exists a safe method in order to study and understand the functions of the physical quantities within the Universe. This method dates back to the Ionian Intellectual Revolution [19], continues through the work of Plato and Aristotle, revives in the great Byzantine and Islamic Civilization [20, 21], and finally returns to Europe, giving birth to the "small" Renaissance of the 12th century, as well as the Renaissance of the Northern Italian cities in the 14th century [22].

This method is commonly known as the "Scientific" one, and incorporates both metaphysical speculations, as well as methodological and observational ones, where the human inquiry is set to discover and state explicitly, and in a mathematical form, where it is achievable, the Physical Laws of Nature. We can mention that the Hellenic civilization gives birth and cultivates strongly and immanently all these spiritual activities. It gives birth to Philosophy, in the form it is known within the realm of the European civilization, to the science of Mathematics, to the science of Astronomy, to the Physical Sciences, to Medicine, to Biology, to Botany, to Metallurgy, to Engineering, and especially to the science of the design and construction of Mechanisms [23], as artifacts and devices, and to the Cartography of Earth and the Sky, just to mention the most basic ones. All these newly- founded, or even newly – introduced sciences were revived as knowledge

and practices in all the great civilizations that followed, certainly inspired by the spirit and the achievements of the Alexandrian era of science [24].

Science requires a certain mentality, a certain vision and aspect towards Nature and Man, as well as the existence of generations of Scholars and Polymaths, and scientific centers, such as Institutes, Academies, Observatories, and Universities, for the transmission of knowledge through education, and the pure and applied scientific research in many diverse fields of knowledge.

The Hellenistic era is one of the first great epochs for the flourishing and blossom of scientific centers and the production of well – educated scholars and polymaths. In this aspect, it may serve as a Prototype, as a Paradigm which found many imitations in the centuries that followed, even after the elapse of many centuries after the fall and the decline of the Roman empire.

Science, as a very special and unique intellectual enterprise, occurs in many forms, while it becomes founded on the science of Logic. The science of Logic develops within the Alexandrian era, and is studied intensively by polymaths and scientists, and especially by the Stoic school of philosophy. The structure of Logos, both as human reasoning, as well as the structure of the various existing sciences, is studied in detail, and especially the logical Paradoxes. These Paradoxes still remain the object of study by modern logicians, mathematicians, and philosophers, and are studied in order to secure on well – grounded foundations Logos, which both corresponds to the human intellectual, as well as to the fabric of Cosmos. Man, by his understanding of himself, his place in the Universe, and by obtaining the basic knowledge of the Laws of Nature can assure his success, his happiness and his prosperity.

The science of Mathematics is also further developed and its content expanded, while the branch of Geometry experiences its first and sound foundation, based on the Axiomatic method, while the Axioms are of a self – evident and obvious character, which everyone can judge and inderstand [25]. This is mainly the greatest achievement of Euclid, introduced in his masterpiece, the so – called “Elements”. We can mention that, within the European civilization, this mathematical jewel and monument of the Human spirit, was read almost as often as the Bible [26].

The science of Astronomy, in particular, is also expanded and further developed, and finds its crown within Claudius Ptolemy’s masterpiece, the “Almagest”, an astronomical work which can be regarded as the counterpart of the “Elements” in the realm of Astronomy.

Ptolemy also gives a foundation to the science of Astronomy, based on self – evident Axioms, which everyone can test and argue about, and whose nature is characterized mainly by simplicity and brevity. Then, he continues its study of the Universe, as it is stated within the framework of the Aristotelian cosmological paradigm [27].

The motions of the then known five planets, together with the motions of the Sun, the Earth and the Moon are described by certain combinations of perfect circular motions. We can expect that the introduction of the epistemological doctrine of the perfect circular motions can be considered as the "brotherly idea" of reproducing them by the use of mechanisms built by cyclical gears.

Ptolemy, in order to represent the motions of the celestial bodies as they have been accurately measured up to his epoch, introduces two important concepts, the concept of the eccentric point, and the concept of the equant point, whose existence and properties determine uniquely the orbit of the studied celestial body. He also introduces, in a systematic way, the two important notions of the deferent cycle and the epicycle. Through certain combinations of eccentrics, equants, and systems of deferents and epicycles, he reproduces the orbits of all known celestial bodies, both qualitatively and quantitatively. The most accurate and quantitative reproduction of a physical phenomenon by means of an axiomatic system of thought and by the strict use of mathematical methods, already found in Ptolemy's work, can be considered as one of the pillars of the Scientific method [28].

This method flourishes and expands within the Hellenistic age and the Alexandrian science, and echoes within the centuries to come, as preserved by the scientific tradition of all great civilizations appearing after the end of this important period.

This world – view is exactly Johannes's Kepler world – view. This world – view both comprises the elements of a certain mentality, methodology and observation, as well as all the astronomical details found in Ptolemy's "Almagest" [29]. The "Greatest" of all astronomical works of all ages before Newton's "Principia" served as the only and unique basis for the science of Astronomy in the Byzantine, Islamic and European civilization, and was also transmitted to the Chinese and Indian civilization. Although it was often heavily commented, even criticized, by the generations of astronomers and philosophers after Ptolemy's death, it was always accepted as the unique way of explaining the celestial phenomena, and the fabric of Cosmos, as well [30].

Kepler departs, on his enterprise for unraveling the secrets of the Universe, from exactly this description of the whole corpus of astronomical phenomena. Without such a basis he could have never achieved in his life – long struggle with the motions of the planets around the Sun, and the motion of Moon around Earth. More important, by learning as a student in the University of Tübingen the Aristotelian doctrine about the Universe, and the Ptolemaic astronomical paradigm, he was also cultivated within an atmosphere of a free thinking and scientific spirit, which enabled him to take his own course, led him eventually to his great achievements in Astronomy, grounded his work in Natural Philosophy, or the science of Mechanics, and introduced a new era in the fast – developing European civilization [31].

Democracy, Theatre, Dialectics, Mathematics, Physics

This wide river of intellectual achievements is accompanied by the introduction to the spiritual and social activities of Tragedy, and in general, of the Theatrical Art, of the institution of Democracy, and of the invention of the Dialectical method. We can speculate that these important achievements, that is Democracy, Dialects, the Theatrical Art, and the science of Mathematics obtaining axiomatic foundations, as well as the strong belief in governing Physical Laws of one and only physical Reality, are indeed correlated to each other. We do not suppose that all these activities of the social spheres and culture are unique to the Hellenic civilization, since all these forms also appear in all the other great Civilizations. The Mesopotamian civilization, the Egyptian civilization, the Persian, the Indian, the Chinese civilizations contributed greatly to the evolution of the human spirit, and all of their aspects shape the Global Heritage of our common cosmopolitan culture. It is only that the achievements of the Hellenic civilization are unique, either by transforming before- existing spiritual Forms completely, or by introducing novel ones.

This mentality, this Zeitgeist, can be considered as a kind of Cybernetic System, with well defined content, functions and properties, which dictate the nature of the Universe, the place of Man within Cosmos, the ability of understanding its fabric, and the laws that govern it. This mentality obtains the form of an Autopoietic structure, and as such it travels within the passage of time, through the capricious turns of History, and into the great civilizations. This Autopoietic structure refers to the scientific mentality and practices, and is expressed within the realm of Astronomy, Medicine, within the realm of Technical innovations, within the Physical Sciences, and also in Philosophy.

It seems that the Weltanschauung, which expands and flourishes within the Alexandrian tradition, is saved within the manuscripts, and within the life and work of enlightened personalities, and is transferred from one great civilization to the other, whenever this civilization experiences its acme and cultivates The Fine Arts, the Sciences and Philosophy.

The science of Astronomy as an Autopoietic Structure through Civilizations

We suppose that within certain eras, when a civilization experiences its acme, then it produces structures and forms of social activities, embedded within the social spheres of that civilization. Then, these structures begin to evolve by themselves, served by a certain class of the members of the society, and their evolution and expansion can both be found within the presence of architectural constructs, of institutions of some kind, of the educational system that reproduces this kind of social class, both as a spiritual tradition, as well as the physical existence of the members of this specific class, which convey and develop this tradition.

The tradition of Astronomy, not only as empirical methods and semi – empirical practices, but as a Science with solid foundations, starts in the Hellenic civilization. It continues its evolution through the ages, from the very first assumptions and observations of the celestial phenomena, and finds its closure within the system of Ptolemy.

We observe that the science of Astronomy becomes an Autopoietic structure, that is a social and intellectual activity, which both contains a spiritual tradition, a certain function, theory and methodology, a tradition of observational measurements and records, dedicated specialized scientists that convey and enrich this tradition, as well as a vast collection of written specialized texts that both educate the next generations of astronomers, as well as present and comment the current status of the astronomical theories.

This structure attains its dynamics through the activities of the astronomers, the scholars and the polymaths, for each generation, through the presence and influence of both the written specialized works of reference, as well as the physical presence and work of the scientists, that collect with ever – growing accuracy and insistence their observational astronomical data, or contribute heavily to the developed theoretical models by their abstract research.

More important, this structure “solidifies” its existence by the strict definition of its theoretical and experimental activities, which in the case of Astronomy coincide with the observational ones, as well as the establishment of a certain mentality among its members. This mentality consists of a system of theoretical speculations and models, of empirical activities, and the sense of belonging to a certain tradition, which plays a significant psychological role among these scientists.

Then, this structure is ready to develop through the often unexpected course of History, as an autonomous, self – contained, self – preserving, adequate, and well – founded corpus of practices and ideas, each time the historical and social conditions allow its existence and blossom.

Astronomy, as such, finds its final form within the framework of the Alexandrian science. Then, it continues its historical course through the Byzantine and Islamic civilization, each time these civilizations become socially stable and economically independent and secured. We observe the same scheme to appear within time: scholars are respected, financed by some enlightened patron, and allowed to built their own “private” space of discourse. Institutes and observatories are built, in order to serve the same purposes these served in the Hellenistic era.

The same forms of the basic astronomical observation instruments appear and re – appear, namely all kinds of dioptras, the crosstuff, the backstuff, the sundials, the waterclocks, the armillary spheres, the astrolabes, the forms of the celestial globus, the mechanical universes or forms of mechanical planetariums, specialized analog computing devices, such as the Antikythera Mechanism, sextants, quadrants, parallactic instruments, and mural instruments [32].

All this corpus of instrumentation is already conceived, built and developed in the Alexandrian era. These instruments are being further re – built and refined in the forthcoming generations of astronomers that follow this great epoch. These are exactly the types of astronomical observation instruments Tycho Brahe designs and builts for his famous Observatory, in the island of Hven, in Denmark [33]. With these instruments he obtains in a systematic and precise way, and with the greatest obtainable accuracy of the technological status of his age, most valuable astronomical tables and ephemeredes.

It is very easy to observe that without the existence of Tycho's Brahe measurements, Johannes Kepler would have never obtained his three eponymous laws for the motion of the planets around the Sun. Hence, also in this respect, the Alexandrian tradition shapes dramatically the life and the work of the great astronomer, as it had shaped the life and work of all famous Byzantine, Islamic and European generations of astronomers that preceded his era.

This attempt becomes possible only after the science of Astronomy becomes an autopoietic structure, with a well established tradition, and spreads through Time and Civilizations as a titanic "Meme", until it reaches the era of Johannes Kepler.

The work of Johannes Kepler as a landmark and a novel begin

Surely, the whole face of the world after Kepler's death changes dramatically. His work can be conceived as a landmark. It defines the gradual change of the autopoietic structure of the science of Astronomy, between its generation in the Ionian Renaissance and its completion within the Alexandrian epoch, and the new form it obtains in the centuries to follow after the death of the great astronomer. These centuries contain multiple ages that changed forever the European civilization and our Global heritage. The Age of Enlightenment, the Age of Industrial Revolution, the Age of Sailing, the development of the Mechanistic philosophy, and the subsequent "mechanization of the world – picture", all become possible and through the influence of Kepler's work on his contemporaries, as well as the next generation of Physical Philosophers [34]. It is enough to remember that according to Newton, he "stand on the shoulders of Giants". Among these giants we can mention Archimedes, Galileo, and Johannes Kepler [35].

This dramatic change would have never become possible without the tradition of the Alexandrian science, and especially the science of Astronomy developed within its realm. It is Kepler's genius that encompasses the core of this tradition, develops further its achievements in the fields of Mathematics, Physics, Astronomy, Optics, and then surpasses them for ever. Then, he finds its own place in the Constellation of the Giants of Astronomy, placed beside Hipparchus, Aristarchus of Samos, and Ptolemy.

The "Mysterium Cosmographicum" of Johannes Kepler

Johannes Kepler starts his astronomical work with one of the most beautiful, but also wrong, physical theories of all ages. The title of his work reads as "Mysterium Cosmographicum", that is the "Mystery of Cosmos", and immediately in this we acknowledge his religious, philosophical and scientific roots of his lifelong achievements than belong to the Hellenic, and to the Alexandrian in particular, tradition.

Here, Kepler uses the Platonic Cosmological Paradigm, as it is stated within the most influential Platonic dialogue "Timaeus"[36]. In "Timaeus" Plato offers a way of geometrical definition of the elements that comprise Cosmos, and all its phenomena. Plato dictates that the five Platonic Solids represent the five fundamental elements, out of which all substances are created. These elements are the Earth, the Fire, the Water, and the Air, while the element of Ether, as substantially introduced later by Aristotle, finds also its geometrical counterpart. The five platonic solids are the tetrahedron, the hexahedron (cube), the octahedron, the dodecahedron, and the icosahedron. The element of Earth takes the form of the hexahedron, the element of Fire takes the form of the tetrahedron, the element of Water takes the form of the icosahedron, Air corresponds to the octahedron, and Ether corresponds to the dodecahedron, or "to the Universe as a whole". All the faces of these regular convex polyhedra can be decomposed into two types of triangles, the equilateral and the scalene half- equilateral right - angled triangles. These triangles can be rearranged in many ways, thus we can obtain the transformation of one Element into another, whenever this is possible, and by following strict numerical relations. We can name these triangles as "Plato's quarks", since they built, with all their mathematical geometrical combinations the Elements of Nature, and dictate the quantitative transformation of one element to another.

The Platonic dialogue "Timaeus" served for many centuries as the prototype for the study of the constituting elements of Nature, and its description was fully integrated, as well as used, within the Aristotelian view of Cosmos.

Johannes Kepler departs its intellectual struggle from the study of the five Platonic solids. Since Euclid already proved that there can be only five platonic solids, a proof which might be regarded as the crown of his famous work "Elements", Kepler suspects that these can be used not on the microscopic scale, but on the macroscopic, the cosmological scale. As a strict follower of the Copernican Cosmological Paradigm, he starts to pose physical questions of fundamental importance, questions that are posed in such a way perhaps for the first time into the science of Astronomy.

Kepler wonders why there are only five planets, why their cyclical orbits around the Sun obtain their exact arithmetical values, as stated within Copernicus work "De Revolutionibus" [37], why their velocities obtain distinct and concrete arithmetical values, with their magnitudes already calculated by Copernicus, why there are only six planets within the Universe.

We have to consider that the then known Universe included the Sun, Mercury, Venus, Earth and Moon, Mars, Jupiter and Saturn, as well as the sphere of the fixed stars, a knowledge that was available to the star gazers of the pre – telescopic era of Astronomy.

Kepler answers this question in the most profound way: the Universe attains its particular qualitative and quantitative characteristics, because there exist only five Platonic solids, intercalated the one into the other, in a precise order of appearance. This structure also determines the ratios of their cyclical orbits, as defined in the work of Nicolaus Copernicus.

This very first attempt of Johannes Kepler contains as a sperm all the ideas and the methodology that accompanied him for the rest of his life, whereas it shows in a direct manner the influence he experienced by the Neoplatonic tradition, the Mathematical method, and the scientific spirit in general, with all of them being the “spiritual children” of the Alexandrian science.

Even his mathematical work on the ellipses relies heavily on the epitome of the Hellenic mathematics “Conic Sections” by Apollonius of Perga. He does not only follow a specific tradition that dictates him his own world – view, he also educates his spirit and grounds his astronomical and physical work on all the important scientific texts of his discipline, all of them created by Giants of science within the Hellenistic age.

In this work, the Medieval and Renaissance European understanding of Philosophy, Science, Mathematics, Astronomy, especially as they developed and took their form within the minds of all Alexandrian philosophers, astronomers and polymaths, reaches its limits, as it gets fully integrated and incorporated in his understanding of the Universe, of Man and his place in Cosmos, of the underlying Harmonies and Symmetries that constitute its fabric, all of them of geometrical and number theoretical nature.

It is here where he shall depart from the various doctrines of this long – standing tradition, always influenced by the spirit of science, by the epistemological value of Rationality and Causation in Nature, and by the habit of being and thinking like a free spirit, so much alike to the tradition of Alexandrian Philosophy and Science.

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